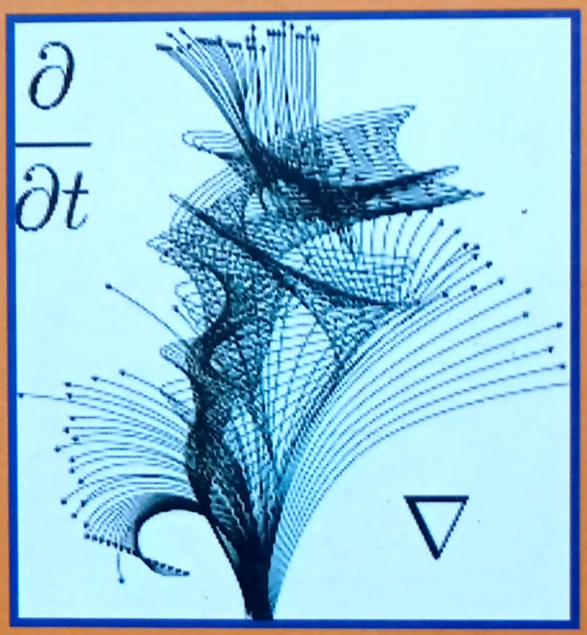
PARTIAL DIFFERENTIAL EQUATIONS

SUDHIR PUJARA • A.K. DAHIYA • JATINDER KAUR • VIKAS POPLY • SANJAY GARG



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PARTIAL DIFFERENTIAL EQUATIONS

For B.A./B.Sc. II Year (Third Semester) Students of M.D.U., K.U. & C.D.L.U.

SUDHIR PUJARA
Department of Mathematics
Govt. College,
BAPAULI

A.K. DAHIYA
Assistant Professor
Department of Mathematics
DAV College, Pehowa
KURUKSHETRA

JATINDER KAUR
Associate Professor
Department of Mathematics
Gurunanak Girls College,
YAMUNA NAGAR (HARYANA)

VIKAS POPLI
Head, Department of Mathematics
K.L.P. College,
Rewari (HARYANA)

Dr. SANJAY GARG
Assistant Professor
Department of Mathematics
R. K. S. D. (PG) College,
KAITHAL

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A.S. PRAKASHAN
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D-106, Saket, Meerut
Phone : 0121-6544643, 6544642
e-mail : asprakashanmrt@gmail.com

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PRAGATI PRAKASHAN
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Preface

This book **'Partial Differential, Equations'** is designed to meet the requirements of B.A./B.Sc.–II (Third semester) students of M.D.U./K.U. & C.D.L.U. The book is equally helpful in the competitive exams. like NET and Civil Services.

This is a self-contained, logically conceived and well-organized book, written with the students of graduation level in mind.

The various concepts, definitions and problems are illustrated with good selected examples and figures.

A serious effort has been made to present the solutions in a smooth and simple manner.

No pains are spared to make the book free from errors. Suggestions and criticism will be warmly received for further improvement of the book.

We are grateful to **'The Almighty'** for providing us the energy to write this work. Finally we express our special thanks to all friends and colleagues for guidance and inspiration.

We are extremely thankful to the Publisher A.S. Prakashan, Meerut for their co-operation and eager efforts for the timely release of this book.

Suggestions for the improvement of the book are expected and solicited.

—Authors

Contents

1. **Partial Differential Equations and their Formation** **1–14**

Introduction 1

Some Important Results and Formulae 2

Notations 3

Derivation of a Partial Differential Equation by the Elimination of Arbitrary Constants 3

Various Situations in the formation of Partial Differential Equation 3

Derivation of Partial Differential Equation by the Elimination of Arbitrary Function ϕ from the Equation $\phi(u, v) = 0$ 9

Exercise 13

2. **First Order Linear Partial Differential Equations** **15–32**

Solution of a Differential Equation 15

Solution of a Linear Partial Differential Equation by Direct Integration 15

Lagrange's Equation 16

Exercise 32

3. **Non-Linear Partial Differential Equations of the First Order** **33–80**

Various Types of Integrals 33

Charpit's Method 34

Jacobi's Method 66

Compatible System of First : Order Equations 73

Exercise 79

4. **Linear Partial Differential Equation of Second and Higher Order** **81–108**

General Linear Partial Differential Equation of Order n 81

Linear Homogeneous Partial Differential of Order n 82

Solution of Homogenous Partial Differential Equation with constant Coefficients 82

Method of Finding the complementary Function of the Linear Homogeneous Partial Differential Equations with constant coefficients 83

Particular Integral 89
Different Methods for finding Particular Integrals 90
General Method of finding the Particular Integrals 92
Method of find P.I. when $f(x, y)$ is of the form x^m, y^n where m and n are integers 104
Exercise 108

5. Non-Homogeneous Linear Partial Differential Equations with Constant Coefficients **109–139**

Method to find the complimentary Function of a Reducible Non-homogenous Linear Partial Differential Equation with Constant Coefficients 109
Working rule for finding C.F. of Reducible Non-homogenous Linear Partial Differential Equation with Constant Coefficients 112
Method of finding C.F. of Irreducible Linear Partial Differential Equation with Constant Coefficients 113
General Solution of non-homogenous Linear Partial Differential Equation with Constant Coefficients 117
Method to Obtain Particular Integral when $f(x, y) = x^m y^n$ 120
Partial Differential Equations with variable co-efficients Reducible to Equations with Constant Co-efficients 129
Exercise 138

6. Classification of Linear Partial Differential Equations **140–169**

Classification of Linear Partial Differential Equation of Second Order in Two Independent Variables 140
Types of Equation 141
Reduction to Canonical Forms 145
Working rule for Reducing a Hyperbolic Equation to its Canonical Form 149
Working rule for Reducing a Parabolic Equation to its Canonical Form 159
Working rule for Reducing an Elliptic Equation to its Canonical Form 165
Exercise 168

7. The Solution of Linear Hyperbolic Equations **170–183**

Riemann Method of Solution of General Linear Hyperbolic Equation of the Second Order 170
Exercise 183

8. Monge's Method **184–223**

Monge's Method of Integrating 184

Monge's Method of Integrating the Equation 207

Exercise 222

9. Characteristic Equations Characteristic Curves and Cauchy's Problems **224–231**

Characteristic Equations and Characteristic Curves 224

Cauchy's Problem for Second Order Differential Equations 228

Exercise 231

10. Method of Separation of Variables : Heat Equation **232–250**

Heat Equation in One and Two dimensions 232

Some Useful Series 232

Method of Separation of Variables 233

Working Procedure to Solve Linear Partial Differential Equation by Separation of Variables Method 233

Superposition Principle 234

Solution of One-dimensional Heat Equation by Method of Separation of Variables 234

Solution of One-dimensional Heat Equation Under Different Boundary Conditions 236

Heat Equation in two Dimensions in Cartesian Co-ordinator 245

Solution of Two-dimensional Heat Equations by Method of Separation of Variables 245

Solution of Two-dimensional Heat Equation Under Given Boundary and Initial Conditions 246

Exercise 250

11. Method of Separation of Variables : Wave Equation **251–269**

Introduction 251

Solution of one-dimensional wave equation by method of separation of variables 251

Solution of one dimensional wave equation under the given boundary and initial conditions 253

Two dimensional wave equation 261

Solution of two dimensional wave equation by method of separation of variables 261

Solution of two dimensional wave equation under the given boundary and Initial conditions by the method of separation of variables 262

Exercise 268

12. Method of Separation of Variables Laplace Equations **270–280**

Laplace Equation 270

Solution of Laplaces equation in two dimensions by method of separation of variables 270

Exercise 280

Short Answer Type Questions **281–284**

Examination Papers